Digital Communication Systems
ECS 452

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7. Additive Noise in the Waveform Channel

Office Hours:

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Monday	14:00-16:00
Thursday	10:30-11:30
Friday	12:00-13:00

Review:

11.5 Linear Dependence

Definition 11.55. Given two random variables X and Y, we may calculate the following quantities:

(a) **Correlation:** $\mathbb{E}[XY]$.

(b) **Covariance:** $\operatorname{Cov}[X,Y] = \mathbb{E}[(X - \mathbb{E}X)(Y - \mathbb{E}Y)]$. $= \mathbb{E}[XY] - (\mathbb{E}X)(\mathbb{E}Y)$

(c) **Correlation coefficient:** $\rho_{X,Y} = \frac{\operatorname{Cov}[X,Y]}{\sigma_X \sigma_Y}$ $= \mathbb{E}\left[\left(\frac{X-\mathbb{E}X}{\sigma_X}\right)\left(\frac{Y-\mathbb{E}Y}{\sigma_Y}\right)\right]$

ECS 315: Probability and Random Processes **2015/1**

HW Solution 10 — Due: Nov 18, 8:59 AM

Lecturer: Prapun Suksompong, Ph.D.

Problem 1 (Randomly Phased Sinusoid). Suppose Θ is a uniform random variable on the interval $(0, 2\pi)$.

(a) Consider another random variable X defined by

$$X = 5\cos(7t + \Theta)$$

where t is some constant. Find $\mathbb{E}[X]$.

(b) Consider another random variable Y defined by

$$Y = 5\cos(7t_1 + \Theta) \times 5\cos(7t_2 + \Theta)$$

where t_1 and t_2 are some constants. Find $\mathbb{E}[Y]$.

Conversion to Vector Channels

Waveform Channel: $R(t) = S(t) + N(t)$

Vector Channel $\vec{R} = \vec{S} + \vec{N}$

Use GSOP to find K orthonormal basis functions $\{\phi_1(t), \phi_2(t), \dots, \phi_K(t)\}$ for the space spanned by $\{s_1(t), s_2(t), \dots, s_M(t)\}$.

This gives vector representations for the waveforms $s_1(t), s_2(t), \dots, s_M(t)$:

$$\vec{s}^{(1)}, \vec{s}^{(2)}, \dots, \vec{s}^{(M)}$$

which can be visualized in the form of signal constellation

Note that $S_j^{(i)}$, the j^{th} component of the $\vec{S}$ vector, comes from the inner-product:

$$S_j^{(i)} = \langle S(t), \phi_j(t) \rangle$$

The received vector $\vec{R}$ is computed in the same way: the j component is given by

$$R_j = \langle r(t), \phi_j(t) \rangle$$

In which case, the corresponding noise vector $\vec{N}$ is computed in the same way: the j component is given by

$$N_j = \langle N(t), \phi_j(t) \rangle$$

Prior Probabilities:

$$p_i = P[W = i] = P\left[S(t) = s_i(t)\right] = P\left[\vec{S} = \vec{s}^{(i)}\right]$$

For additive white **Gaussian** noise (AWGN) process N(t),

$$N_j \sim N \sim \mathcal{N}\left(0, \frac{N_0}{2}\right) = \mathcal{N}\left(0, \sigma^2\right) \Rightarrow \vec{N} \sim \mathcal{N}\left(\vec{0}, \frac{N_0}{2} I\right) \Rightarrow f_{\vec{N}}(\vec{n}) = \frac{1}{(2\pi)^{\frac{K}{2}} \sigma^K} e^{-\frac{1}{2}\frac{\|\vec{n}\|^2}{\sigma^2}}$$